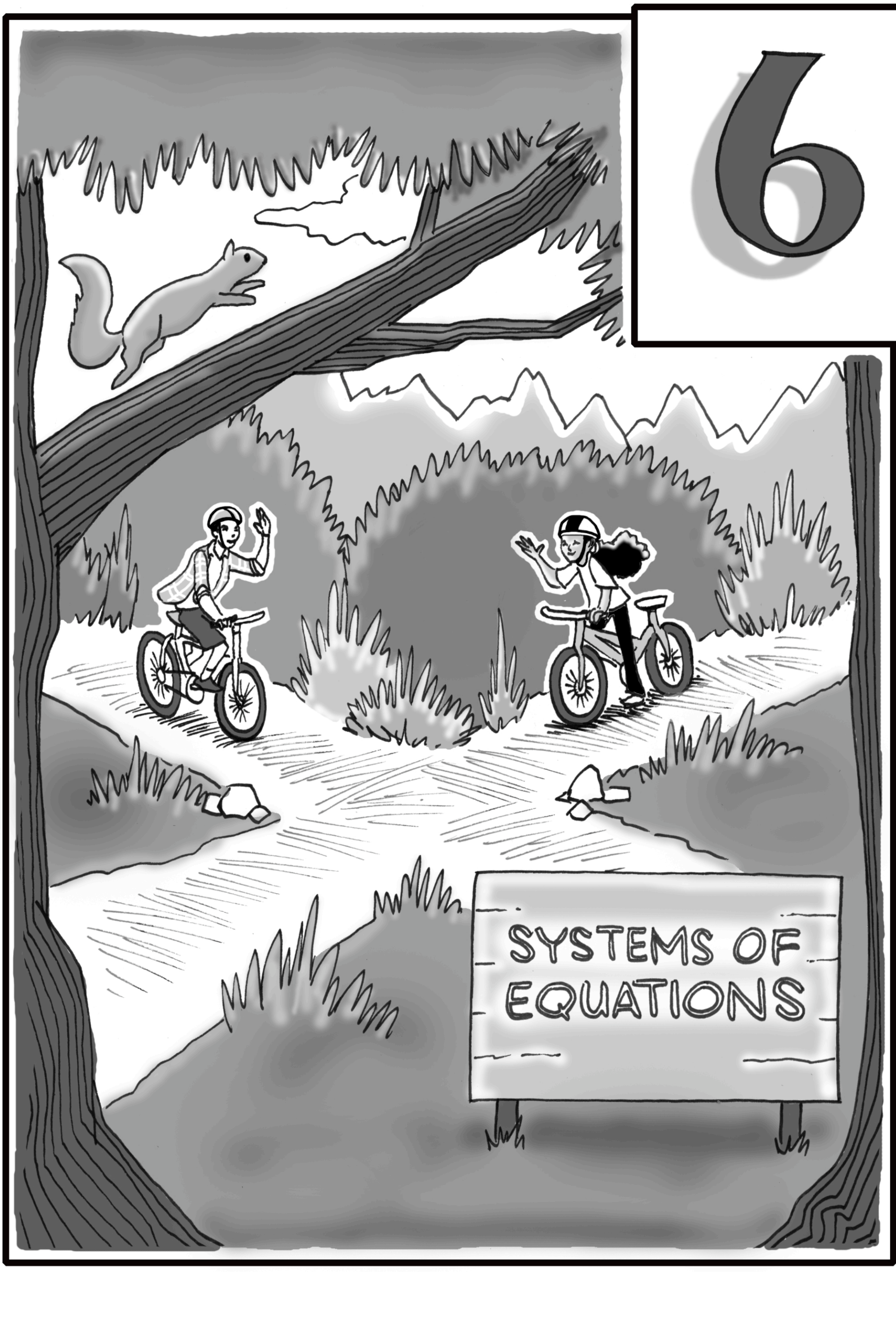


6

CHAPTER 6 Systems of Equations

In Chapter 4, you studied the **connections** between the multiple representations of data and learned how to write equations from situations. You also developed a way to solve a system of equations. In this chapter, you will learn how to solve word problems by writing an equation (or a system of equations). Also, unlike previous chapters, where you were limited to certain kinds of systems of equations, in this chapter you will learn how to solve *any* system of linear equations, regardless of its form.

Along the way, you will develop new ways to solve different forms of systems and will learn how to recognize when one method may be most efficient. By the end of this chapter, you will know multiple ways to find the point of intersection of two lines and will be able to solve systems that arise from different contexts.

Guiding Questions

Think about these questions throughout this chapter:

What is a solution?

How can I represent it algebraically?

How can I solve it?

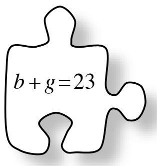
Is there another way?

How can I check my answer?

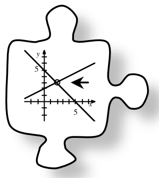
In this chapter, you will learn:

- What a solution of a system of equations represents.
- How to solve contextual word problems by writing and solving equations.
- How to recognize systems of equations that have no solution or infinite solutions.
- How to solve different forms of systems quickly and efficiently.

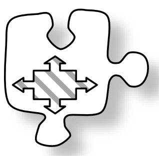
Chapter Outline



Section 6.1 In this section, you will write and solve mathematical sentences (such as one- and two-variable equations) to solve contextual word problems.

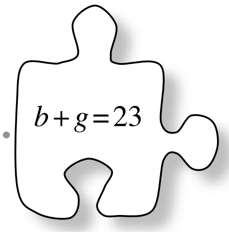


Section 6.2 You will develop methods to solve systems of equations in different forms. You will learn which equations will result in lines when graphed. You will also find ways to know which solving method is most efficient and accurate.



Section 6.3 Section 6.3 provides an opportunity for you to review and **extend** what you learned in Chapters 1 through 6. You will make important **connections** between solving equations, multiple representations, proportional reasoning, and systems of equations.

6.1.1 How can I write it using algebra?



Mathematical Sentences

Spoken and written languages use *sentences* to convey information. A sentence has a subject and verb, follows the rules of grammar, and is structured with punctuation. Likewise, algebra uses **mathematical sentences**, such as $b + g = 23$, which also convey information and follow structural rules.

During this lesson, you will explore various mathematical sentences and learn how to interpret their meanings. Then you will write mathematical sentences of your own.

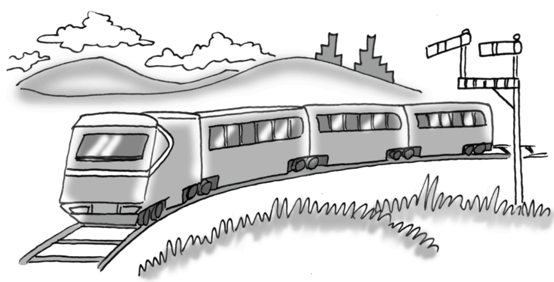
6-1. How can variables give you new information? Suppose Mr. Titelbaum's class has b boys and g girls.

a. Mr. Titelbaum noticed that $b + g = 23$. What does that tell you about his class?

b. If $b = g - 3$, what statement can you make about the number of boys and girls?

c. How many girls are in Mr. Titelbaum's class? Explain how you know.

6-2. The local commuter train has three passenger cars. When it is sold out, each passenger car can hold p people.



a. In addition to the passengers, the train has 8 employees. Write an expression that represents the total number of people on this commuter train.

b. When it is sold out, the train has a total of 176 people on board. Write an equation that represents this fact.

c. Solve your equation to determine how many people a passenger car can hold. Be sure to check your solution when you are finished.

6-3. MATHEMATICAL SENTENCES

A **mathematical sentence** uses variables and mathematical operations to represent information. For example, if you know that b represents the number of boys in a class and g represents the number of girls in the same class, then the mathematical sentence "$b + g = 23$" states that if you add the number of boys to the number of girls, you get a result of 23. In other words, there are 23 students in the class.

Mathematical Sentence:	b	$+$	g	$=$	23	
Same Sentence in English:	*The number of boys*	*plus*	*the number of girls*	*is*	*23 students.*	

While many mathematical sentences contain more than one variable (such as $b + g = 23$ above), some only contain one variable. For example, if p represents the maximum number of people in a train's passenger car, then the mathematical sentence $3p + 8 = 176$ states that a train with 3 passenger cars and 8 additional people will have 176 people in all. This is shown below.

Mathematical Sentence:	$3p$	$+$	8	$=$	176
Same Sentence in English:	*Three passenger cars*	*plus*	*8 employees*	*is*	*176 people.*

Mathematical sentences convey information once you understand what each variable represents. Sometimes the structure of the equation or the letter of the variable can reveal its possible meaning. With your team, study the two mathematical sentences below and decide what each could be trying to communicate. Be prepared to share your description with the class.

a. $0.25q + 0.05n = 5.00$

b. $l + w + l + w = 30$

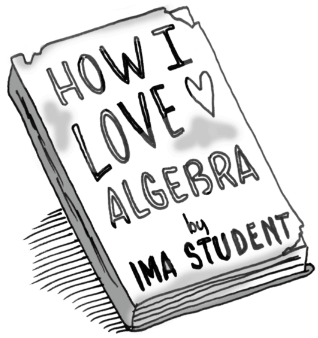
6-4. Mathematical sentences are easier to understand when everyone knows what the variables represent. For example, if you knew that l in part (b) of problem 6-3 represented the length of one side of a rectangle, then it would have been easier to understand that the mathematical sentence $l + w + l + w = 30$ could have been stating that the perimeter of a rectangle is 30 units.

A statement that describes what the variable represents is called a **"let" statement**. It is called this because it often is stated in the form "Let $l = \ldots$". While solving the problems below, examine how "let" statements are used.

a. Let m = the number of students at Mountain View High School and let $m - 100$ = the number of students at neighboring Ferguson High School. Which school has more students? How can you tell?

b. Based on the "let" statements in part (a) above, translate this mathematical sentence into English: $m + (m - 100) = 5980$.

c. A book called *How I Love Algebra* has only three chapters. Let p = the number of pages in Chapter 1, $p + 12$ = the number of pages in Chapter 2, and $\frac{p}{2}$ = the number of pages in Chapter 3. Which is the longest chapter? Which is the shortest?



d. Using the definitions in part (c) above, write and solve a mathematical sentence that states that *How I Love Algebra* has 182 pages. How many pages are in Chapter 1?

6-5. With your team, practice translating words into mathematical symbols. For each problem below, write an expression or equation that best represents the given situation.

a. Turner rode his bike m miles. If Carolyn rode 10 less than twice the number of miles that Turner rode, how many miles did Carolyn ride?

b. Your teacher spent \$9.50 on 5 boxes of chalk and 2 boxes of overhead pens. If c represents the price of a box of chalk and p represents the price of a box of overhead pens, write an equation to represent this purchase.

c. Each fruit basket comes with a apples, p pears, and b bananas. Wendi orders 4 fruit baskets and gets 84 pieces of fruit. Write an equation that represents this order.

6-6. In your Learning Log, write your own mathematical sentence. Be sure to state what any variables represent. Title this entry "Writing Mathematical Sentences" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Mathematical Sentences

A **mathematical sentence** uses variables and mathematical operations to represent information. An equation is one type of mathematical sentence. When the variables are defined, a mathematical sentence can be translated into a sentence with words.

For example, if *b* represents the number of boys in a class and if *g* represents the number of girls, then the mathematical sentence $b + g = 23$ states that the total number of boys and girls is 23.

Mathematical Sentence:	b	$+$	g	$=$	23
Same Sentence in English:	*The number of boys*	*plus*	*the number of girls*	*is*	*23.*

Review & Preview

6-7. Solve the problem below using a Guess and Check table. ***Note:*** *Be sure to put your work in a safe place, because you will need it for the next lesson.*

The perimeter of a triangle is 31 cm. Sides #1 and #2 have equal length, while Side #3 is one centimeter shorter than twice the length of Side #1. How long is each side?

6-8. Write expressions to represent the quantities described below.

a. If Thompson Valley High School has x students and if Erwin Middle School has 342 fewer students, how many students does Erwin Middle School have?

b. If w represents the width of a rectangle and if its length is twice its width, how long is the rectangle?

c. When Mr. Van Exel bought his laptop, he paid \$400 more than three times the amount he paid for his camera. If he paid c dollars for his camera, then how much did he pay for his laptop?

6-9. Solve the system of equations below using the Equal Values Method.

$$a = 12b + 3$$
$$a = -2b - 4$$

6-10. Ms. Cai's class is studying a tile pattern. The rule for the tile pattern is $y = 10x - 18$. Kalil thinks that Figure 12 of this pattern will have 108 tiles. Is he correct? **Justify** your answer.

6-11. Angel is picking blackberries in her backyard for a delicious pie. She can pick 9 blackberries in 2 minutes. If she needs 95 blackberries for the pie, how long will it take her to pick the berries?

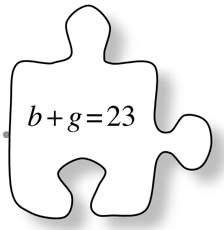
6-12. Juan thinks that the graph of $6y + 12x = 4$ is a line.

a. Solve Juan's equation for y.

b. Is this equation **linear**? That is, is its graph a line? Explain how you know.

c. What are the growth factor and y-intercept of this graph?

6.1.2 How can I use variables to solve problems?



Solving Word Problems by Writing Equations

In Lesson 6.1.1, you examined mathematical sentences (equations that convey related information). Today you will learn more ways to translate written information into algebraic symbols and will then solve the equations that represent the relationships.

6-13. Match each mathematical sentence on the left with its translation on the right.

a. $2z+12=30$

b. $12z+5(z+2)=30$

c. $z+(z-2)+5(z-2)=30$

d. $z+12z=30$

1. A zoo has two fewer elephants than zebras and five times more monkeys than elephants. The total number of elephants, monkeys, and zebras is 30.

2. Zola earned \$30 by working two hours and receiving a \$12 bonus.

3. Thirty ounces of metal is created by mixing zinc with silver. The number of ounces of silver needed is twelve times the number of ounces of zinc.

4. Eddie, who earns \$5 per hour, worked two hours longer than Zach, who earns \$12 per hour. Together they earned \$30.

6-14. In Lesson 6.1.1, you examined how to translate words into mathematical symbols to form expressions and equations. However, you can also use Guess and Check tables to help you write mathematical sentences. Find your solution for problem 6-7, reprinted below.

The perimeter of a triangle is 31 cm. Sides #1 and #2 have equal length, while Side #3 is one centimeter shorter than twice the length of Side #1. How long is each side?

a. Add a row to your Guess and Check table. If x represents the length of Side #1, then what is the length of Side #2? Side #3? Fill in the columns for Sides #1, #2, and #3 with these variable expressions.

b. Write a mathematical sentence that states that the perimeter is 31 cm.

c. If you have not done so already, solve the equation you found in part (b) and determine the length of each side. Does this answer match the one you got for problem 6-7?

6-15. For the following word problems, write one or two equations and then solve the problem. You may choose to use a Guess and Check table to help you set up equations, although it is not required. Regardless of your method, be sure to define your variable(s) with appropriate "let" statements.

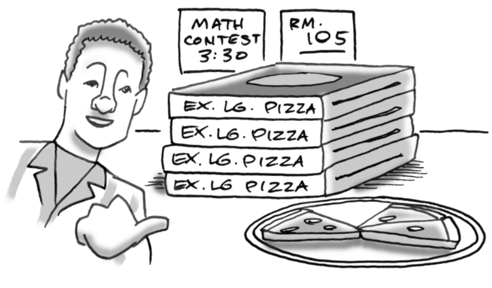
a. Herman and Jacquita are each saving money to pay for college. Herman currently has $15,000 and is working hard to save $1000 per month. Jacquita only has $12,000 but is saving $1300 per month. In how many months will they have the same amount of savings?

b. There are 21 animals on Farmer Cole's farm – all sheep and chickens. If the animals have a total of 56 legs, how many of each type of animal lives on his farm?

c. When ordering supplies, Mr. Williams accidentally ordered 12 more than twice his usual number of pencils. When the order arrived, he received 60 pencils! How many pencils does Mr. Williams usually order?

d. George bought some CDs at his local store. He paid $15.95 for each CD. Nora bought the same number of CDs from a store online. She paid $13.95 for each CD, but had to pay $8 for shipping. In the end, both George and Nora spent the exact same amount of money buying their CDs! How many CDs did George buy?

e. After the math contest, Basil noticed that there were four extra-large pizzas that were left untouched. In addition, another three slices of pizza were uneaten. If there were a total of 51 slices of pizza left, how many slices does an extra-large pizza have?



Review & Preview

6-16. Solve for x. Check your solutions, if possible.

a. $-2(4-3x)-6x=10$

b. $\frac{x-5}{-2}=\frac{x-1}{-3}$

6-17. On the same set of axes, graph the two rules shown at right. Then find the point(s) of intersection, if one (or more) exists.

$y=-x+2$
$y=3x+6$

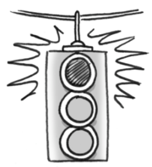
6-18. Evaluate the expression $6x^2-3x+1$ for $x=-2$.

6-19. The basketball coach at Washington High School normally starts each game with the following five players:

Melinda, Samantha, Carly, Allison, and Kendra

However, due to illness, she needs to substitute Barbara for Allison and Lakeisha for Melinda at this week's game. What will be the starting roster for this upcoming game?

6-20. When Ms. Shreve solved an equation in class, she checked her solution and it did not make the equation true! Examine her work below and find her mistake. Then find the correct solution.

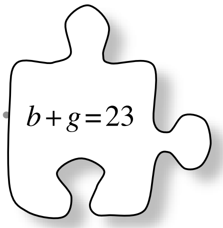


$$5(2x-1)-3x=5x+9$$
$$10x-5-3x=5x+9$$
$$7x-5=5x+9$$
$$12x=4$$
$$x=\frac{1}{3}$$

6-21. Determine if the statement below is true or false. **Justify** your conclusion.

$$2(3+5x)=6+5x$$

6.1.3 How can I solve the system?



Solving Problems by Writing Equations

In Lessons 6.1.1 and 6.1.2, you created mathematical sentences that represented word problems. But how can you tell if you can use one variable or two? And is one method more convenient than another? Today you will compare the different ways to represent a word problem with mathematical symbols.

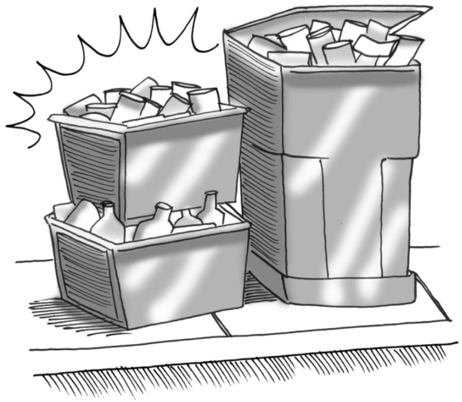
You will also explore how to use the Equal Values Method to solve systems containing equations that are not in $y = mx + b$ form.

6-22. ONE EQUATION OR TWO?

Review what you learned in Lesson 6.1.2 by answering the questions below.

a. Solve the problem below using Guess and Check.

Elsie took all of her cans and bottles from home to the recycling plant. The number of cans was one more than four times the number of bottles. She earned 10¢ for each can and 12¢ for each bottle, and ended up earning \$2.18 in all. How many cans and bottles did she recycle?



b. Use your Guess and Check table to help you write an equation that represents the information in part (a). Be sure to define your variable.

c. If you have not done so already, solve your equation from part (b). Does this solution match your answer to part (a)? If not, look for and correct any errors.

d. How can this problem be represented using two variables? With your team, write two mathematical sentences that represent this problem. Be sure to state what your variables represent. You do not need to solve the system.

e. Show that your solution from part (a) makes both equations in part (d) true.

6-23. Renard thinks that writing two equations for problem 6-22 was easy, but he's not sure if he knows how to solve the system of equations. He wants to use two equations with two variables to solve this problem:

> Ariel bought several bags of caramel candy and taffy. The number of bags of taffy was 5 more than the number of bags of caramels. Taffy bags weigh 8 ounces each, and caramel bags weigh 16 ounces each. The total weight of all of the bags of candy was 400 ounces. How many bags of candy did she buy?

a. Renard lets t = the number of taffy bags and c = the number of caramel bags. Help him write two equations to represent the information in the problem.

b. Now Renard is stuck. He says, "If both of the equations were in the form 't = something,' I could use the Equal Values Method to find the solution." Help him change the equations into a form he can solve.

c. Solve Renard's equations to find the number of caramel and taffy bags that Ariel bought. Check to make sure your solution works.

6-24. When you write equations to solve word problems, you sometimes end up with two equations like Renard's or like the system shown at right. Notice that the second equation is solved for y, but the first is not. Change the first equation into $y = mx + b$ form, and then solve this system of equations. Discuss with your team how you can make sure your solution is correct.

$$2y + 8x = 10$$
$$y = 5x + 23$$

6-25. Solve each system below by first changing each equation so that it is in $y = mx + b$ form. Check that your answer makes both equations true.

a. $x - 2y = 4$
$y = -\frac{1}{2}x + 4$

b. $x + 2y = 14$
$-x + 3y = 26$

Review & Preview

6-26. Write expressions to represent the quantities described below.

a. Geraldine is 4 years younger than Tom. If Tom is t years old, how old is Geraldine? Also, if Steven is twice as old as Geraldine, how old is he?

b. 150 people went to see "Ode to Algebra" performed in the school auditorium. If the number of children that attended the performance was c, how many adults attended?

c. The cost of a new CD is $14.95, and the cost of a video game is $39.99. How

6-27. Nina has some nickels and 9 pennies in her pocket. Her friend, Maurice, has twice as many nickels as Nina. Together, these coins are worth 84¢. How many nickels does Nina have? Solve using any method, but show all of your work.

6-28. To count the number of endangered falcons in the local county, Fernando first tagged each of the 8 falcons he saw one day. Then, days later, he counted 11 falcons and noticed that only 3 were tagged. What is a good estimate of how many falcons exist in his county? Show how you know.

6-29. As Sachiko solved the equation $(x+2)+3=9$, she showed her work in the table below. Copy the table and provide justification for each step.

Statement	**Reason**
1. $(x+2)+3=9$	Given
2. $x+(2+3)=9$	
3. $x+5=9$	
4. $x+5-5=9-5$	
5. $x=4$	

6-30. A **prime number** is defined as a number with exactly two integer factors: itself and 1. Jeannie thinks that all prime numbers are odd. Is she correct? If so, state how you know. If not, provide a counterexample.

6-31. In an "If…then…" statement, the "if" portion is called the **hypothesis**, while the "then" portion is called the **conclusion**. For example, in the statement "*If* $x=3$, *then* $x^2=9$," the hypothesis is "$x=3$" while the conclusion is "$x^2=9$."

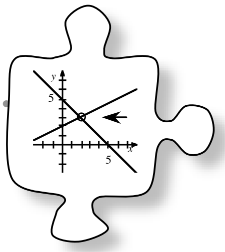
Identify the hypothesis and conclusion of each of the following statements. Then decide if you think the statement is true or not. **Justify** your decision.

a. If $-x=8$ then $x=-8$.

b. If $3x+y=-11$, then $6x+2y=-22$.

c. If Tomas runs at a constant rate of 4 meters every five seconds, then he will run 50 meters in 1 minute.

6.2.1 How can I solve the system?



Solving Systems of Equations Using Substitution

In Chapter 4, you learned that a set of two or more equations that go together is called a **system of equations**. In Lesson 6.1.3, you helped Renard develop a method for solving a system of equations when one of the equations was not solved for a variable. Today you will develop a more efficient method of solving systems that are too messy to solve with the Equal Values Method.

6-32. Review what you learned in Lesson 6.1.3 as you solve the system of equations below. Check your solution.

$$\begin{aligned} y &= -x-7 \\ 5y+3x &= -13 \end{aligned}$$

6-33. AVOIDING THE MESS

A new method, called the **Substitution Method**, can help you solve the system in problem 6-32 without getting involved in messy fractions. This method is outlined below.

a. If $y=-x-7$, then does $-x-7=y$? That is, can you switch the y and the $-x-7$? Why or why not?

$$\begin{aligned} y &= -x-7 \\ 5y+3x &= -13 \end{aligned}$$

b. Since you know that $y=-x-7$, can you switch the y in the second equation with $-x-7$ from the top equation? Why or why not?

$$\begin{aligned} y &= -x-7 \\ 5\,y+3x &= -13 \end{aligned}$$

c. Once you replace the y in the second equation with $-x-7$, you have an equation with only one variable, as shown below. This is called **substitution** because you are substituting for (replacing) y with an expression that it equals. Solve this new equation for x and then use that result to find y in either of the original equations.

$$5(-x-7)+3x=-13$$

6-34. Use the Substitution Method to solve the systems of equations below.

a. $y = 3x$
$2y - 5x = 4$

b. $x - 4 = y$
$-5y + 8x = 29$

c. $2x + 2y = 18$
$x = 3 - y$

d. $c = -b - 11$
$3c + 6 = 6b$

6-35. When Mei solved the system of equations below, she got the solution $x = 4$, $y = 2$. *Without solving the system yourself*, can you tell her whether this solution is correct? How do you know?

$$\begin{aligned} 4x + 3y &= 22 \\ x - 2y &= 0 \end{aligned}$$

6-36. HAPPY BIRTHDAY!

You've decided to give your best friend a bag of marbles for her birthday. Since you know that your friend likes green marbles better than red ones, the bag has twice as many green marbles as red. The label on the bag says it contains a total of 84 marbles.

How many green marbles are in the bag? Write an equation (or system of equations) for this problem. Then solve the problem using any method you choose. Be sure to check your answer when you are finished.

Review & Preview

6-37. Solve each equation for the variable. Check your solutions, if possible.

a. $8a + a - 3 = 6a - 2a - 3$

b. $8(3m - 2) - 7m = 0$

c. $\frac{x}{2} + 1 = 6$

d. $4t - 2 + t^2 = 6 + t^2$

6-38. The Fabulous Footballers scored an incredible 55 points at last night's game. Interestingly, the number of field goals was 1 more than twice the number of touchdowns. The Fabulous Footballers earned 7 points for each touchdown and 3 points for each field goal.

a. **Multiple Choice:** Which system of equations below best represents this situation? Explain your reasoning. Assume that t represents the number of touchdowns and f represents the number of field goals.

i. $t = 2f + 1$
$7t + 3f = 55$

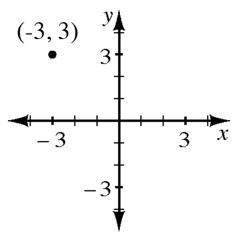
ii. $f = 2t + 1$
$7t + 3f = 55$

iii. $t = 2f + 1$
$3t + 7f = 55$

iv. $f = 2t + 1$
$3t + 7f = 55$

b. Solve the system you selected in part (a) and determine how many touchdowns and field goals the Fabulous Footballers earned last night.

6-39. Yesterday Mica was given some information and was asked to find a linear equation. But last night her cat destroyed most of the information! At right is all she has left:



x	y
–3	
–2	1
–1	
0	
1	
2	
3	

a. Complete the table and graph the line that represents Mica's rule.

b. Mica thinks the equation for this graph could be $2x + y = -3$. Is she correct? Explain why or why not. If not, find your own algebraic rule to match the graph and $x \rightarrow y$ table.

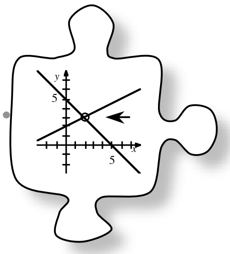
6-40. Kevin and his little sister, Katy, are trying to solve the system of equations shown below. Kevin thinks the new equation should be $3(6x - 1) + 2y = 43$, while Katy thinks it should be $3x + 2(6x - 1) = 43$. Who is correct and why?

$$y = 6x - 1$$
$$3x + 2y = 43$$

6-41. Create a table and graph the rule $y = 10 - x^2 + 3x$. Label its x- and y-intercepts.

6-42. Maurice thinks that $x = -2$ is a solution to $x^2 - 3x - 8 = 0$. Is he correct? Explain.

6.2.2 How does a graph show a solution?

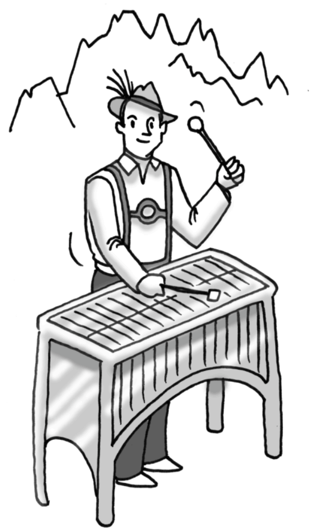


Making Connections: Systems, Solutions, and Graphs

In this chapter you have practiced writing mathematical sentences to represent situations. Often, these sentences give you a system of equations, which you can solve using substitution. Today you will start to represent these situations in an additional way: on a graph. You will also examine more closely what makes a solution to a two-variable equation.

6-43. THE HILLS ARE ALIVE

The Alpine Music Club is going on its annual music trip. The members of the club are yodelers, and they like to play the xylophone. This year they are taking their xylophones on a gondola to give a performance at the top of Mount Monch.



The gondola conductor charges \$2 for each yodeler and \$1 for each xylophone. It costs \$40 for the entire club, including the xylophones, to ride the gondola. Two yodelers can share a xylophone, so the number of yodelers on the gondola is twice the number of xylophones.

How many yodelers and how many xylophones are on the gondola?

Your Task:

- Represent this problem with a system of equations. Solve the system and explain how its solution relates to the yodelers on the music trip.
- Represent this problem with a graph. Identify how the solution to this problem appears on the graph.

Discussion Points

How can the given information be represented with equations?

What is a solution to a two-variable equation?

How can this problem be represented on a graph?

How does the solution appear on the graph?

Further Guidance

6-44. Start by focusing on one aspect of the problem: the cost to ride the gondola. The conductor charges \$2 for each yodeler and \$1 for each xylophone. It costs \$40 for the entire club, with instruments, to ride the gondola.

a. Write an equation with two variables that represents this information. Be sure to define your variables.

b. Find a combination of xylophones and yodelers that will make your equation from part (a) true. Is this is the only possible combination?

c. List five additional combinations of xylophones and yodelers that could ride the gondola if it costs \$40 for the trip. With your team, decide on a good way to organize and share your list.

d. Jon says, "I think there could be 28 xylophones and 8 yodelers on the gondola." Is he correct? Use the equation you have written to explain why or why not.

e. Helga says, "Each correct combination we found is a *solution* to our equation." Is this true? Explain what it means for something to be a solution to a two-variable equation.

6-45. Now consider the other piece of information: The number of yodelers is twice the number of xylophones.

a. Write an equation (mathematical sentence) that expresses this piece of information.

b. List four different combinations of xylophones and yodelers that will make this equation true.

c. Put the equation you found in part (a) together with your equation from problem 6-44 and use substitution to solve this system of equations.

d. Is the answer you found in part (c) a solution to the first equation you wrote (the equation in part (a) of problem 6-44)? How can you check? Is it a solution to the second equation you wrote (the equation in part (a) of this problem)? Why is this a solution to the *system* of equations?

6-46. The solution to "The Hills are Alive" problem can also be represented graphically.

a. On graph paper, graph the equation you wrote in part (a) of problem 6-44. The points you listed for that equation may help. What is the shape of this graph? Label your graph with its equation.

b. Explain how each point on the graph represents a solution to the equation.

c. Now graph the equation you wrote in part (a) of problem 6-45 on the same set of axes. The points you listed for that equation may help. Label this graph with its equation.

d. Find the intersection point of the two graphs. What is special about this point?

e. With your team, find as many ways as you can to express the solution to "The Hills are Alive" problem. Be prepared to share all the different forms you found for the solution with the class.

Further Guidance section ends here.

6-47. Consider this system of equations:

$$2x + 2y = 18$$
$$y = x - 3$$

a. Use substitution to solve this system.

b. With your team, decide how to fill in the rest of the table at right for the equation $2x + 2y = 18$.

x	***y***
–2	11
–1	
0	
1	
2	
3	

c. Use your table to make an accurate graph of the equation $2x + 2y = 18$.

d. Now graph $y = x - 3$ on the same set of axes. Find the point of intersection.

e. Does the point of intersection you found in part (a) agree with what you see on your graph?

6-48. If you had an equation with three variables, how would you write its solutions?

6-49. What is a solution to a two-variable equation? Answer this question in complete sentences in your Learning Log. Then give an example of a two-variable equation followed by two different solutions to it. Finally, make a list of all of the ways to represent solutions to two-variable equations. Title your entry "Solutions to Two-Variable Equations" and label it with today's date.

MATH NOTES

METHODS AND MEANINGS

The Substitution Method

The **Substitution Method** is a way to change two equations with two variables into one equation with one variable. It is convenient to use when only one equation is solved for a variable.

For example, to solve the system:

$$x = -3y + 1$$
$$4x - 3y = -11$$

Use substitution to rewrite the two equations as one. In other words, replace *x* with $(-3y+1)$ to get $4(-3y+1) - 3y = -11$. This equation can then be solved to find *y*. In this case, *y* = 1.

To find the point of intersection, substitute to find the other value.

Substitute $y = 1$ into $x = -3y + 1$ and write the answer for *x* and *y* as an ordered pair.

To test the solution, substitute $x = -2$ and $y = 1$ into $4x - 3y = -11$ to verify that it makes the equation true. Since $4(-2) - 3(1) = -11$, the solution must be correct.

$$x = -3y + 1$$
$$4(\quad) - 3y = -11$$
$$4(-3y+1) - 3y = -11$$
$$-12y + 4 - 3y = -11$$
$$-15y + 4 = -11$$
$$-15y = -15$$
$$y = 1$$
$$x = -3(1) + 1 = -2$$
$$(-2,\ 1)$$

Review & Preview

6-50. Camila is trying to find the equation of a line that passes through the points (–1, 16) and (5, 88). Does the equation $y = 12x + 28$ work? **Justify** your answer.

6-51. Solve the systems of equations below using the method of your choice. Check your solutions, if possible.

a. $y = 7 - 2x$
$2x + y = 10$

b. $3y - 1 = x$
$4x - 2y = 16$

6-52. Hotdogs and corndogs were sold at last night's football game. Use the information below to write mathematical sentences to help you determine how many corndogs were sold.

a. The number of hotdogs sold was three fewer than twice the number of corndogs. Write a mathematical sentence that relates the number of hotdogs and corndogs. Let h represent the number of hotdogs and c represent the number of corndogs.

b. A hotdog costs \$3 and a corndog costs \$1.50. If \$201 was collected, write a mathematical sentence to represent this information.

c. How many corndogs were sold? Show how you found your answer.

6-53. Examine the balanced scales in Figures 1 and 2 shown below. Figure 1 shows that two candies balance three dice. Figure 2 shows that one rubber ball balances two jacks.

Figure 1

Figure 2

Figure 3

Determine what could be placed on the right side of the scale in Figure 3 to balance with the left side. **Justify** your solution in complete sentences.

6-54. Rianna thinks that if $a = b$ and if $c = d$, then $a + c = b + d$. Is she correct?

6-55. For each of the following generic rectangles, find the dimensions (length and width) and write the area as the product of the dimensions and as a sum.

a.

$3y^2$	$-12y$

b.

$3y^2$	$-12y$
$5y$	-20